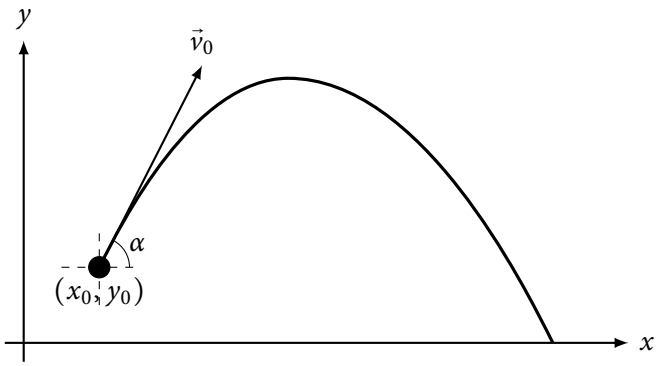
Lesson 10. Projectile Motion

1 In this lesson...

- Describing the trajectory of a projectile with parametric equations

2 Trajectory of a projectile

- A projectile with mass m is fired
  - initial point (x_0, y_0)
  - angle of elevation α
  - initial velocity $\vec{v}_0$
- Assume:
  - Air resistance is negligible
  - The only external force is due to gravity



- Let's derive parametric equations that describe the trajectory of this projectile

1. Let's define $v_0 = |\vec{v}_0|$ (we're just renaming the initial speed, or the magnitude of the initial velocity). Using this new notation, write $\vec{v}_0$ in terms of v_0 and α. *Hint.* We'll need to use trigonometry.

2. We need an expression for the acceleration $\vec{a}(t)$ of the projectile.

   Recall Newton's second law of motion: if at any time t, a force $F(t)$ acts on an object of mass m producing an acceleration $\vec{a}(t)$, then $\vec{F}(t) = m\vec{a}(t)$.

   Since the only external force is due to gravity, which acts downward, we have that $\vec{F}(t) = m\vec{a}(t) = \langle 0, -mg \rangle$. Solve for $\vec{a}(t)$.

3. Using our answer from part 2, write an expression for the velocity $\vec{v}(t)$ of the projectile.

   Hint 1. Recall that $\vec{a}(t) = \vec{v}'(t)$. *Hint 2.* Don't forget the constant vector of integration. *Hint 3.* Since the initial velocity is $\vec{v}_0$, we have $\vec{v}(0) = \vec{v}_0$. Use the expression for $\vec{v}_0$ we obtained in part 1.

4. Now, using our answer from part 3, write an expression for the position $\vec{r}(t)$ of the projectile.

   Hint 1. Recall that $\vec{v}(t) = \vec{r}'(t)$. *Hint 2.* Don't forget the constant vector of integration. *Hint 3.* Since the initial point is (x_0, y_0), we have $\vec{r}(0) = \langle x_0, y_0 \rangle$.

5. Expand the vector equation we obtained in part 4 to write parametric equations (i.e. $x = \ldots, y = \ldots$) for the trajectory of the projectile.

3 Problems

In each of these problems, ignore the possibility of air resistance. Assume that acceleration due to gravity is downward and equal to g.

Problem 1. A cannon sitting atop of a 200 m cliff shoots a projectile at a speed of 50 m/s and at an angle of 30° above the horizontal. A building 50 m tall sits 300 m from the base of the cliff. Does the projectile strike the building? (Ignore the width of the building).

Problem 2. A lacrosse player 80 m from an open goal throws a ball at an angle of 25° above the horizontal with a speed of 20 m/s. Does the ball enter the goal in the air? Assume that the ball leaves the stick 3 m above the ground and that a lacrosse goal is 2 m high.

Problem 3. An F-18 is flying at 200 m/s at an altitude of 1500 m and at an angle of 5° below the horizontal when it drops a bomb. There is a 300 m building 3000 m from the point below the F-18 when it drops its bomb. Does the bomb hit the building? (Ignore the width of the building.)

Problem 4. In Fenway Park, the Green Monster is a wall approximately 11.3 m tall and 94 m from home plate along third base line. A ball was hit at an angle of 30° along the third base line and barely cleared the Green Monster. At what speed did the ball leave the bat? Assume that batter hit the ball 1 m above the ground.